Different Approaches to Cointegration Analysis of Economic Time Series with Applications to German Price Index Series and Ukrainian Price Series

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Analysis of Economic and Environmental Time Series

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A) Introduction of the EURO in Germany

- After introduction of euro notes and coins in January 2002, there was a widespread feeling of a significant hike in consumer prices in Germany.
- Substantial discrepancy between a) inflation measured by the consumer price index (CPI) and b) the one perceived by the general public.
- There is clearly a special inflation around introduction of the Euro.
- The IPI is about 4% higher than the CPI after 2002.
In 2002, the word teuro was voted word of the year.

The official statistics retorted that euro is no teuro.

The gap between perceived and measured inflation is an important phenomenon [Del Giovane and Sabbatini, 2005].

H. W. Brachinger outlines a theory of perceived inflation:

1. Prices of goods are coded as gains or losses with respect of a reference price,
2. These gains and losses are evaluated by a value function,
3. The inflation perception is influenced by the frequency of perceived gains resp. losses.
A) Introduction of the EURO in Germany

- EURO or ‘TEURO’?

![](index_stands_vpi_iwi.png)

- Analysis of perception of inflation versus inflation on the basis of the Consumer Price Index
- Cointegration analyses gives the tools to justify the linear relationships between the series.
A) Cointegration period: October 2001 - July 2005

- WGI = wahrgen. Inflation (EU Consumer Survey)
- IWI = Index der wahrgenommenen Inflation

Cointegration of German Price Indexes

\[ \hat{IWI}_{t-9} = 0.101 WGI_t + 0.922 \quad , \quad t = 2001 : 10 - 2005 : 7 \]
A) Coint. period: January 2001 - October 2004

- WGI = Wahrgen. Inflation (EU Consumer Survey)
- IWl = Index der wahrgenommenen Inflation

\[ \hat{WGI}_{t+9} = 9.894 IWI_t - 9.123 \quad , \quad t = 2001 : 1 - 2004 : 10 \]
B) Ukrainian transition economy: Data base

- Data from the State Committee of Statistics of Ukraine (Derzhkomstat)
- 19 monthly basic food price time series
- *Consumer price index* (CPI) $\sim$ Price level $\sim$ price of a basket of goods
- The prices of 1992 have to be multiplied by a factor of 5000 to get the prices of 2002
- *Mean total wages* (MTW) of the Ukrainian labor force

- The new currency has been introduced on September 2, 1996

![Figure 2. Official exchange rate of hryvnia against one US dollar for the period 1992:12-2001:12](source: NBU)

- Since the year 2000, the stability is only apparent, there are great fluctuations in the prices
B) Similarity: CPI and MTW

Abbildung: Ukrainian consumer price index and mean total wages
The notion of cointegration (simplified):

1. Two components $y_{tj}, y_{ti}$ of the vector $y_t = [y_{t1}, \ldots, y_{t21}]'$ are said to be cointegrated of order 1, $y_t \sim CI(1)$, if

2. The components $y_{tj}, y_{ti}$ are integrated $I(1)$ of order 1 (first differences of each component are stationary).

3. There exists a vector $\beta = [\beta_j, \beta_i]'$ such that the linear combination $\beta_j y_{tj} + \beta_i y_{ti} = \varepsilon_t \sim I(0)$ is stationary.

4. or: There exists a vector $\beta = [\beta_j, \beta_i]'$ such that the linear combination $\beta_j y_{tj} + \beta_i y_{ti} + \alpha = \varepsilon_t \sim I(0)$ is stationary.

5. All the computations are performed with RATS and CATS (Estima, Illinois, USA) and with PCGIVE-OX (Timberlake, UK).
Andreou and Spanos (2003) proposed to analyse the statistical adequacy or the empirical validity of probabilistic assumptions underlying a statistical model.

Spanos set up a traditional AR(k) model

\[ y_t = \alpha + \mu t + \Psi D_t + \sum_{j=1}^{p} \phi_j y_{t-j} + \varepsilon_t. \]  (3)

Spanos requests in his statistical adequacy analyses to look for five properties either of the data or resulting in the residuals: (1) Normality, Skewness and Kurtosis of the process \( y_t \), (2) Linearity, (3) Homoskedasticity, (4) \( t \)-invariance of the parameters, (5) Martingale difference process of the residuals.
B) Unit root tests: \( \Rightarrow \text{I}(1) \)

- The Schmidt-Phillips unit root test (1992)
- Deterministic trend modelled as a polynomial \( f_n(t) \)

<table>
<thead>
<tr>
<th>Nr.</th>
<th>Indexes</th>
<th>( y_t ) ( p = 1 )</th>
<th>( y_t ) ( p = 2 )</th>
<th>( y_t ) ( p = 3 )</th>
<th>( \Delta y_t ) ( p = 1 )</th>
<th>( \Delta y_t ) ( p = 2 )</th>
<th>( \Delta y_t ) ( p = 3 )</th>
<th>Int.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bread</td>
<td>-0.86</td>
<td>-1.18</td>
<td>-1.28</td>
<td>-3.38</td>
<td>-5.13</td>
<td>-5.12</td>
<td>I(1)</td>
</tr>
<tr>
<td>2</td>
<td>Flour</td>
<td>-1.26</td>
<td>-1.43</td>
<td>-1.40</td>
<td>-7.07</td>
<td>-7.11</td>
<td>-7.10</td>
<td>I(1)</td>
</tr>
<tr>
<td>3</td>
<td>Macaroni</td>
<td>-1.08</td>
<td>-1.17</td>
<td>-1.19</td>
<td>-8.59</td>
<td>-8.54</td>
<td>-8.53</td>
<td>I(1)</td>
</tr>
</tbody>
</table>

**Tabelle:** Schmidt-Phillips root tests with polynomial degrees \( p = 1, 2, 3 \)
B) Unit root tests: $\Rightarrow I(1)$

- The Bhargava-type unit root test (1986)
- RATS computes the $R_1$- (random walk against stationarity), $R_2$-statistics (random walk with drift against stationarity)

<table>
<thead>
<tr>
<th>Indexes</th>
<th>$y_t$</th>
<th>$y_t$</th>
<th>$\Delta y_t$</th>
<th>$\Delta y_t$</th>
<th>$\Delta^2 y_t$</th>
<th>$\Delta^2 y_t$</th>
<th>Int.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_1$</td>
<td>$R_2$</td>
<td>$R_1$</td>
<td>$R_2$</td>
<td>$R_1$</td>
<td>$R_2$</td>
<td></td>
</tr>
<tr>
<td>Bread</td>
<td>0.0044</td>
<td>0.0335</td>
<td>0.6049</td>
<td>0.049</td>
<td>1.9808</td>
<td>1.9808</td>
<td>I(1)</td>
</tr>
<tr>
<td>Flour</td>
<td>0.0062</td>
<td>0.0403</td>
<td>0.9757</td>
<td>0.9757</td>
<td>2.5534</td>
<td>2.3213</td>
<td>I(1)</td>
</tr>
<tr>
<td>Macaroni</td>
<td>0.0031</td>
<td>0.0254</td>
<td>1.2974</td>
<td>1.2881</td>
<td>2.9379</td>
<td>2.9047</td>
<td>I(1)</td>
</tr>
</tbody>
</table>

**Tabelle:** Sargan-Bhargava unit root tests
The nature of economic time series

- George Udny Yule, 1871-1951: pay attention on spurious regressions (1926)!
- Ch. R. Nelson and Ch. I. Plosser (1982): most *macroeconomic time series* have unit roots!
- J.H. Cochrane (1988): mixture of *random walk* and *unit roots*
- A. Harvey (1997): Structural breaks are best modelled with *Kalman filters*
B) Graphics of prices of goods

Ukrainian food prices

- Bread
- Manka
- Butter
- Flour
- Beef
- Eggs
- Macaroni
- Buckwheat
- Rice

Hryvna

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Power Purchase Analysis
B) CPI, MTW and Graphics of prices

Consumer price index, total wages, Ukrainian food prices

J.-F. Emmenegger  Power Purchase Analysis
In economics, purchasing power refers to the amount of goods and services a given amount of money or, more generally, liquid assets can buy.

1) What quantity of given basic foods could a representative household or individual purchase with the earning power of its labour, represented by the mean monthly total wages, at any cross-sectional time point within the decade 1993-2000, respectively 2000-2004?

Did the situation improve from 1993-2000 to 2000-2004?

2) Was the price level, measured by the CPI, in equilibrium with the mean total wages MTW within the decade 1991-2004?

Consider the pairs: $(MTW; y_{ti})$ and $(MTW; CPI)$.

The purchasing power is presented by the affinity eq.

$$q_i = \frac{y_{t21}}{y_{ti}} \sim \text{const.}, i = 1, ..., 19, t = 1, .., T \quad (4)$$

or written as

$$y_{t21} - q_i y_{ti} = 0, i = 1, ..., 19, t = 1, .., T \quad (5)$$

Indeed, the equations (5) are nothing but a *regression between time series* (!).
Cointegration analysis starts with setting up a $p-$order vector autoregressive $VAR(p)$ model

$$y_t = \alpha + \beta t + \Psi D_t + \sum_{j=1}^{p} A_j y_{t-j} + \varepsilon_t ; \quad t = 1, \ldots, T \quad \varepsilon_t \sim n.i.i.d(0, \Sigma),$$

(6)

Are the intercepts $\alpha = 0$ and the trends $\mu = 0$?

$$y_t = \Psi D_t + \sum_{j=1}^{p} A_j y_{t-j} + \varepsilon_t ; \quad t = 1, \ldots, T \quad \varepsilon_t \sim n.i.i.d(0, \Sigma),$$

(7)
B) Vector autoregression (VAR)

- Cointegration analysis: Is it possible to get rid of the intercepts and the trends? No constant is a necessary condition for purchasing power analysis!
- The Akaike Information Criterion (AIC) and a Likelihood-Ratio (LR) test are applied model

<table>
<thead>
<tr>
<th></th>
<th>TotWages and (1)</th>
<th>AIC with $\alpha, \mu$ (2)</th>
<th>AIC without $\alpha, \mu$ (3)</th>
<th>LR $\chi^2(4)$</th>
<th>$p - value$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bread</td>
<td>-358.57</td>
<td>-357.04</td>
<td>4.809980</td>
<td>0.307356</td>
<td></td>
</tr>
<tr>
<td>Flour</td>
<td>-215.10</td>
<td>-210.61</td>
<td>7.418164</td>
<td>0.115373</td>
<td></td>
</tr>
<tr>
<td>Macaroni</td>
<td>-215.39</td>
<td>-216.24*</td>
<td>2.752509</td>
<td>0.600058</td>
<td></td>
</tr>
<tr>
<td>Manka</td>
<td>-291.31</td>
<td>-288.45</td>
<td>5.995038</td>
<td>0.199519</td>
<td></td>
</tr>
<tr>
<td>Beef</td>
<td>90.59</td>
<td>90.10</td>
<td>3.073304</td>
<td>0.545634</td>
<td></td>
</tr>
<tr>
<td>Buckwheat</td>
<td>114.66</td>
<td>114.19</td>
<td>5.705401</td>
<td>0.222256</td>
<td></td>
</tr>
</tbody>
</table>

**Tabelle:** Likelihood-Ratio test for reduction of $\alpha$ and $\mu$ in the VAR(2)-model of Total Wages and one of the series of column (1)
Vector-Error correction model

- Simple algebra brings a VAR representation (7) equivalently into a VECM

\[
\Delta y_t = \Psi D_t + A y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + \varepsilon_t,
\]

(8)

- Consequently, with \( y_t = [y_{t1}, y_{ti}]', i = 1, \ldots, 19 \) the cointegration relation (5) is stationary.

\[
Ay_t = \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
y_{t2} \\
y_{ti}
\end{bmatrix} = \begin{bmatrix}
a_{11} y_{t2} + a_{12} y_{ti} \\
a_{21} y_{t2} + a_{22} y_{ti}
\end{bmatrix} = \begin{bmatrix}
\varepsilon_{t2} \\
\varepsilon_{ti}
\end{bmatrix}
\]

(9)

\[
y_{t21} = q_{ti} y_{ti} + \varepsilon_{ti} \quad \varepsilon_{ti} \sim iid(0, \sigma_i^2), i = 1, \ldots, 19.
\]

(10)
### B) $\lambda_{trace}$-test for the rank $\rho$, Ljung-Box residual’s test

<table>
<thead>
<tr>
<th>Wages and (1)</th>
<th>E.V. (2)</th>
<th>$\lambda_{trace}$ (3)</th>
<th>$\lambda_{trace}$ (4)</th>
<th>$k$ (5)</th>
<th>$LB$ (6)</th>
<th>$LM(i)$ (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bread 1993:10-1998:10</td>
<td>0.167</td>
<td>12.450</td>
<td>12.282</td>
<td>4.071</td>
<td>50</td>
<td>85.452(0.001)</td>
</tr>
<tr>
<td>Bread 2001:12-2004:9</td>
<td>0.262</td>
<td>12.547</td>
<td>12.282</td>
<td>4.071</td>
<td>22</td>
<td>33.520(0.055)</td>
</tr>
<tr>
<td>Bread 1993:10-2001:10</td>
<td>0.128</td>
<td>14.635</td>
<td>12.282</td>
<td>4.071</td>
<td>86</td>
<td>142.72(0.000)</td>
</tr>
<tr>
<td>Macaroni 1993:10-2001:01</td>
<td>0.173</td>
<td>18.356</td>
<td>12.282</td>
<td>4.071</td>
<td>78</td>
<td>156.75(0.000)</td>
</tr>
<tr>
<td>Macaroni 2003:06-2004:11</td>
<td>0.529</td>
<td>15.166</td>
<td>12.282</td>
<td>4.071</td>
<td>10</td>
<td>13.645(0.190)</td>
</tr>
<tr>
<td>Macaroni 1997:10-2001:01</td>
<td>0.249</td>
<td>12.412</td>
<td>12.282</td>
<td>4.071</td>
<td>30</td>
<td>33.567(0.295)</td>
</tr>
<tr>
<td>CPI 1993:10-1998:12</td>
<td>0.212</td>
<td>12.528</td>
<td>12.282</td>
<td>4.071</td>
<td>50</td>
<td>48.797(0.522)</td>
</tr>
<tr>
<td>CPI 1996:12-2002:12</td>
<td>0.240</td>
<td>19.556</td>
<td>12.282</td>
<td>4.071</td>
<td>62</td>
<td>165.09(0.000)</td>
</tr>
</tbody>
</table>
Cointegration \{\text{rank}=1, \text{lags}=2, \text{noseasons}\} of Bread and MTW

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Power Purchase Analysis
B) Bread and Mean Total Wages

Cointegration \{\text{cirank}=1,\text{lags}=2,\text{noseans}\} of Bread and MTW

Bread and MTW, period 1993:10 1998:10

Bread and MTW, period 1996:10 2002:1

Bread and MTW, period 1993:10 2001:10

Bread and MTW, period 2001:12 2004:9

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Power Purchase Analysis
B) Macaroni and Mean Total Wages

Cointegration \{\text{rank}=1, lags=2, \text{no seasons}\} of Macaroni and MTW

- $W_{X31}$
- $W_{X32}$
- $W_{X33}$
- $W$

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Power Purchase Analysis
B) Macaroni and Mean Total Wages

Cointegration \{cirank=1, lags=2, no seaosns\} of Macaroni and MTW

Macaroni and MTW, period 1993:10 2001:1

Macaroni and MTW, period 2003:6 2004:12

Macaroni and MTW, period 1997:10 2001:1

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Power Purchase Analysis
B) CPI and Mean Total Wages

Cointegration \{rank=1, lags=2, noseasons\} of CPI and MTW

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Power Purchase Analysis
B) CPI and Mean Total Wages

Cointegration \{\text{cirank}=1, \text{lags}=2, \text{noseasons}\} of CPI and MTW

CPI and MTW, period 1993:10 1998:10

CPI and MTW, period 1996:10 2000:12

CPI and MTW, period 1998:01 2000:12

CPI and MTW, period 1996:12 2002:12

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Power Purchase Analysis
B) Purchasing Power in different intervals

<table>
<thead>
<tr>
<th></th>
<th>period (2)</th>
<th>( \beta_i ) (3)</th>
<th>period (4)</th>
<th>( \beta_i ) (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2003:08-2004:11</td>
<td>197</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- rank of cointegration in all these cases: \( \rho = 1 \)
A) B) Conclusions

- Cointegration analysis
  1. G: Lagged lin. transf. exist between IWI and WGI
  2. U: Purchasing Power relation is an affine transf.

- Economic Statements
  1. Germany: The perceived inflation is 4% higher than measured inflation since 2002.
  2. Ukraine: The consumer price index and the mean total wages show strong cointegration only in sub-periods between 1993 and 2004.
  3. Ukraine: The evolution of the cointegration factors $\beta_i$ between some price time series of basic food and mean total wages in sub-periods of 1993-2004 show that the purchasing power is actually increasing.

- The analyses show a problem of non-linearity! [⇒ Dufrénot and Mignon: nonlinear cointegration! ]