THE ISBA BULLETIN

Vol. 11 No. 1 *March 2004*

The official bulletin of the International Society for Bayesian Analysis

A MESSAGE FROM THE NEW PRESIDENT

by Jim Berger
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Being an ISBA member is the best deal I know of in the world. Bayesians are a diverse lot, being spread throughout virtually all areas of science and society, and staying connected to each other is difficult. ISBA has set out to be the connecting fiber, and is being highly successful in this regard. Let me remind you of why.

First, there are our meetings. We are enormously excited about our 2004 World Meeting, scheduled from the 23rd to 27th of May, 2004 in Viña Del Mar, Chile. The program, prepared by the Scientific Committee chaired by Fabrizio Ruggieri, is absolutely stellar, and the venue and local arrangements put in place by the Local Organizing Committee, chaired by Pilar Iglesias and Fernando Quintana, will make for the wonderful ambience that is the tradition of major Bayesian meetings. We owe a lot to these three individuals and those on their committees, as well as to Alicia Carriquiry, as chair of the Finance Committee. The meeting will also serve as the happy occasion for award of the second Degroot Prize, for an outstanding book in statistical science, and the inaugural award of the Lindley prize, for the outstanding contributed paper at the last Valencia/ISBA-World meeting.

I also hope to see you at the other upcoming ISBA cosponsored or endorsed meetings: the *Bayesian Nonparametrics - IV Workshop*, to be held June 13-16, 2004 in Rome, Italy; the *International Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering (MaxEnt04)*, to be held July 25-30, 2004 in Garching bei Mnchen, Germany; the *International Conference on Bayesian Statistics and its Applications*, to be held January 6-8, 2005 in Varanasi, India; and *MCMski: the Past, Present and Future of Gibbs Sampling*, the 2nd IMS-ISBA joint meeting, to be held January 12-14, 2005 in Bormio, Italy. The second great tie-in for Bayesians that is provided by ISBA is its Chapters. The current Chapters in Chile, India and South Africa are thriving, as is the new chapter in Brazil, which held the 7th Brazilian Meeting for Bayesian Statistics from February 8-11, 2004; I unfortunately was unable to attend the meeting, but heard from those who were there that it was a great success. We also are looking forward to the formation of new chapters, such as a proposed Australian chapter.

In my list of wonderful current reasons to belong to ISBA, I have left one of the best to last, namely the Bulletin. This is by far my favorite 'read' of any society newsletter, and we are highly indebted to Hedibert Freitas Lopes for his efforts in continuing the great tradition of the ISBA Bulletin.

ISBA is about to become even better. Coming this summer (Northern hemisphere summer) is the long-awaited new ISBA electronic journal *Bayesian Analysis*. This will be a new home for innovative research about Bayesian theory, methodology and application. It will serve as a highly visible centerpiece for ISBA, a magnet that will attract the very best in Bayesianism worldwide. We are incredibly fortunate to have Rob Kass leading this effort as Editor-In-Chief; he has assembled a stunning editorial board and the upcoming first issues of the journal are going to be masterpieces of Bayesian research, application and exposition.

During my year as president, I hope to establish another avenue for the worldwide integration of Bayesianism, namely the creation of sections of ISBA. A number of long-standing and highly prominent Bayesian groups exist in the world, such as the MaxEnt community (see, e.g., the MaxEnt conference mentioned above). These groups have their own scientific traditions and activities that are to be treasured. Still, communication between the groups could be greatly enhanced if they participated through a central organization, such as ISBA. Finding mechanisms to enable such communication is a win/win situation for all Bayesians.

I want to give a hearty welcome to our incoming officers: President-Elect Sylvia Richardson and Executive Secretary Deborah Ashby; and to our new Board Members, Brad Carlin, Merlise Clyde, David Higdon, and David Madigan. I very much look forward to working with you this year.

Finally, ISBA has grown into the present dynamic organization through the very considerable efforts of many, of whom I want to acknowledge a few that have been especially central to its development this past year and whose terms have ended. These include Past-President David Draper, Past-Chair of the Program Council Tony O'Hagan, and former Board members Nicky Best, Eduardo Gutiérrez-Peña, Tony O'Hagan and Raquel Prado; all these individuals are stepping down after three years of devoted service. Someone who is not stepping down, but whom I'll thank nevertheless, is Ed George; this past year he has done an incredible amount for ISBA (and Bayesian analysis in general), but luckily I will have the benefit of another year of his wisdom while he serves on the executive committee as Past-President.

A MESSAGE FROM THE EDITOR

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Firstly, I would like to thank Lilla Di Scala and Luca La Rocca for their magnificent term ahead of the Student's Corner section for the last two years. Luca has now graduated (see his thesis title and abstract below) and soon Lilla is leaving us. They have worked hard and have always looked for new insights to improve the section. Wherever you go guys, just keep up the good work that things will turn out just right. My very best regards to you both.

Secondly, I would like to thank, in the name of the ISBA Bulletin team, Ed George for his explicit and crucial support for the 10th volume of the Bulletin during his 2004 presidency.

Finally, Let us welcome Jim Berger, the ISBA president for 2004. For the record, I first met Jim at a gathering in David Higdon's house back in 1997, right after he moved to Durham, North Carolina. Of course I "met" his book much earlier when I was studying for my Master's qualifying exam in Brazil. Anyway, we graduate students were extra excited to meeting him personally. I remember that a few hours and beers later, we were trying to elicit the curve that describes one's satisfaction towards pure tequila or cachaça, on one hand, and Margerita or caipirinha, on the other hand. It has always been a great learning experience to read, talk, chat, and listen to Jim. I look forward to work under your presidential supervision this year.

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PLEASE, FEEL COMPLETELY FREE TO SEND US SUGGESTIONS THAT MIGHT IMPROVE THE QUALITY OF THE BULLETIN

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Numerical Integration is an Art, Not a Science

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This is an extended summary of Kuonen (2003), where current quadrature methods for approximate calculation of integrals within σor R were reviewed. The aim of this survey paper was to help readers, not expert in computing, to apply numerical integration methods and to realize that numerical integration is an art, not a science. Herein we only give a short overview of the art of numerical integration within statistics.

For the available implementations within σor R, for further details on the mentioned techniques and for the results of an extensive comparison study we refer the interested reader to Kuonen (2001, 2003).

1 Introduction

Numerical integration, also called *quadrature*, is the study of how the numerical value of an integral can be found. All quadrature methods are based, in one way or another, on the obvious device of adding up the value of the integrand at a sequence of points within the range of integration. Hence, most of the approximations have the form

$$\int \cdots \int_{R_m} w(x_1, \ldots, x_m) f(x_1, \ldots, x_m) dx_1 \cdots dx_m \approx \sum_{i=1}^{M} W_i f(y_{i,1}, \ldots, y_{i,m}), \quad (1)$$

where R_m is a given region in a m-dimensional Euclidean space E_m and $w(x_1, \ldots, x_m)$ is a given weight function. The $(y_{i,1}, \ldots, y_{i,m})$ lie in E_m and are called the *points* of the formula. The W_i are constants which do not depend on $f(x_1, \ldots, x_m)$ and are called the *coefficients* of the formula. We say that formula (1) has a *degree of exactness* r if it is exact for all polynomials in $x_1, \ldots, x_m$ of degree $\leq r$ and there is at least one polynomial of degree $r+1$ for which it is not exact.

The theory of integration formulae for functions of one variable ($m = 1$) is well developed; see Davis and Rabinowitz (1984) or Evans (1993). The integral of a function is approximated by the sum of its values at a set of equally spaced points, multiplied by certain aptly chosen coefficients of the formula. Examples include the trapezoidal and Simpson's rules. Hence only the W_i are free to be used to force the quadrature rule to have a certain degree of exactness. The freedom to fix the points y_i has been thrown away, presumably in the interests of getting linear equations for the W_i. If the y_i are also left free, the result is a set of non-linear equations which can be shown to have solutions based on the zeros of the associated sets of orthogonal polynomials for the given interval $[a, b]$ and weight function $w(x)$. This leads to the elegant theory of Gaussian quadrature.

2 Gaussian quadrature

The idea is to give ourselves the freedom to choose not only the coefficients W_i, but also the location of the points at which the function is to be evaluated. Moreover, the integration formula is forced to have a certain degree of exactness, *e.g.* $2M - 1$. Because of the computational expense of generating a new Gaussian formula, only commonly used combinations of the interval and weight functions are normally tabulated. The most commonly used rule is the Gauss–Legendre rule with interval $[-1, 1]$ and weight function $w(x) = 1$. To apply it to any finite range quadrature on the interval $[a, b]$, the linear transformation

$$x = \frac{b-a}{2} t + \frac{b+a}{2}$$

to the standard interval $[-1, 1]$ can be used.

For the multi-dimensional case we reduce the multiple integral on the left-hand side of (1) into repeated integrals over $[-1, 1]$,

$$\int_{-1}^{1} dx_1 \int_{-1}^{1} dx_2 \cdots \int_{-1}^{1} f(x_1, \ldots, x_m) dx_m. \quad (2)$$

Then we apply a classical quadrature formula to each integral in (2), which yields using the right-hand side of (1) a product rule of the form

$$\sum_{i_m=1}^{M} \cdots \sum_{i_1=1}^{M} W_{i_1} \cdots W_{i_n} f(y_{i_1}, \ldots, y_{i_m}), \quad (3)$$

where the weights W_{i_j} and the points y_{i_j}, $j = 1, \ldots, m$, are chosen to be appropriate for the specific dimension to which they are applied.

For M quadrature points in each dimension the sum in (3) is over M^m terms. Therefore the numerical effort of Gaussian quadrature techniques increases exponentially with the integral dimension.

Furthermore, the trouble with Gaussian quadrature is that you have no real idea of how accurate the answer is. You can always increase the accuracy by using a higher order Gauss method or by applying it piecewise over smaller periods but you still do not know the accuracy in terms of correct decimal places. To get a prescribed accuracy one needs adaptive integration, which keeps reducing the step size until a specified error has been achieved.

3 Adaptive methods

Indeed, Gaussian quadratures are formulae which are said to be *progressive*, as the points for any point-number M are in general quite different from those for any other point-number. Another term used to describe quadrature rules is *adaptive*. A rule is adaptive if it compensates for a difficult subrange of an integrand by automatically increasing the number of quadrature points in the awkward region. Adaptive rules are usually based on a standard underlying quadrature rule, often a progressive one. Adaptive algorithms are now used widely for the numerical calculation of multiple integrals. Genz (1992) suggested that such subregion adaptive integration algorithms can be used effectively in some multiple integration problems arising in statistics. The key to good solutions for these problems is the choice of an appropriate transformation from the infinite integration region for the original problem to a suitable finite region for the subregion adaptive algorithm. Care must be taken in the selection of the transformation. As a check on consistency and efficiency we encourage use of several transformations for different computations of the same integral, and then comparison of their results.

4 Monte Carlo methods

Numerical methods known as Monte Carlo (MC) methods can be loosely described as statistical simulation methods. For a complete introduction to MC integration we refer to Robert and Casella (1999, chap 3). The basic MC method iteratively approximates a definite integral by uniformly sampling from the domain of integration, and averaging the function values at the samples. The integrand is treated as a random variable, and the sampling scheme yields a parameter estimate of the mean, or expected value of the random variable. However, MC methods suffer from a slow convergence. To reduce the error, for example, by a factor of 10 requires a 100-fold increase in the number of sample points. Therefore, other methods have been studied for decreasing the error. Such approximations are called 'Quasi Monte Carlo' (QMC) methods, where the points used for evaluating the function are generated deterministically. The resulting accuracy of the integral is generally significantly better than in the MC method. Many different QMC methods are known. One method makes use of results from the the theory of numbers and is called the *number-theoretic* method; see Fang and Wang (1994) or Fang *et al.* (1994).

Additional methods have been employed to reduce the error of the MC method, such as importance sampling, stratified sampling, antithetic variates and non-random sequences (see Evans and Swartz, 1995, for a review). These methods are mostly concerned with finding sets of points that yield smaller integration errors. Importance sampling concentrates samples in the area where they are more effective by using *a priori* knowledge of the function. Stratified sampling tries to distribute samples evenly by subdividing the domain into subregions such as grids. It is possible to combine some of these techniques, or to apply them adaptively. For example, uniformly distributed samples generated by stratification can be employed for importance sampling.

As described above, MC integration draws samples from the required distribution, and then forms sample averages to approximate expectations. 'Markov Chain Monte Carlo' (MCMC) methods draw these samples by running a constructed Markov chain for a long time. An example of a way to construct such a chain is the Gibbs sampler. An introduction to MCMC methods and their applications is given in Gilks *et al.* (1996) or Robert and Casella (1999). But questions on convergence of the chains and efficient implementation remain unresolved (Cappé and Robert, 2000). Moreover, we do not feel so comfortable using the MC methods mentioned for an additional reason: one needs too many function evaluations to get a certain accuracy.

5 Discussion

Traditional quadrature methods (even newer adaptive ones) have been almost forgotten in the recent rush to MCMC methods. Genz and Kass (1997) argued that the reason why existing quadrature methods have been largely overlooked in statistics may be that when applied they are poorly suited for peaked-integrand functions.

Indeed, the numerical integration routine samples the function at a number of points, and then assumes that the function varies smoothly between these points. As a result, if none of the sample points come close to the peak, then it will go undetected, and its contribution will not be correctly included. Therefore it is very important to get an idea of the effective range of the integrand in a preliminary analysis. But, if such a problem is thought to have arisen, one could bypass these problems using the split-t transformations proposed in Genz and Kass (1997) prior to the use of adaptive numerical integration algorithms.

Finally, one should not forget that numerical integration in statistics, especially in multidimensional problems still raises many open questions.

Further information on the numerical computation of multiple integrals in statistics may be found in Evans and Swartz (2000) and in the references given in Kuonen (2001, 2003).

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VII Brazilian Meeting of Bayesian Statistics: A summary report

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The VII Brazilian Meeting of Bayesian Statistics, endorsed and cosponsored by ISBA, was organized by ISBRA, the brazilian chapter of ISBA, and by the Statistics Department of the Universidade de São Carlos, in São Paulo, Brazil. The meeting was held last February, 8th-11th, in the Hotel Anacã. Around 110 participants contributed to this successful, diverse and rather multinational meeting: 101 participans from Brazil, 5 from the US, 2 from Chile, one from Canadá and one from Italy. A tutorial on *Nonparametric Bayesian Data Analysis* was ministred by Peter Müller, while 6 conference talks were given by Peter Müller, Joseph Kadane, Jim Zidek, Liseo Brunero, Jorge Ashcar and Carlos Pereira. Sessions on *Statistics in Finance*, *Environmental Statistics, and Skewed Distributions*, a *young researcher* session and 57 posters completed the meeting. The best four posters, chosen by the participants, will appear in the second issue of the *Boletim ISBRA*, a brazilian analog of the ISBA bulletin. We have to acknowledge Josemar Rodrigues, and the organizing committee, for making the meeting possible and smooth. Keep your eyes open because the VIII meeting will be held in Rio de Janeiro, probably somewhere between October/2005 and March/2006, or in other words, summer in Rio!